Managing Variety for Assembled Products: Modeling Component Systems Sharing

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Component sharing—using the same version of a component across multiple products—is an approach adopted by many assembled-product manufacturers to achieve high final product variety with lower component variety and cost. This paper presents a methodology for determining which versions of a set of related components should be offered to optimally support a defined finished product portfolio. We develop optimization models that determine which versions of each component should be introduced and which of these versions each product should use to minimize design and production costs. This approach is appropriate for components with a relatively low impact on consumers’ perceptions about product differentiation, which can be shared across a set of products if they meet the most stringent performance requirements in the set. We illustrate our procedure on automotive braking systems, but also discuss its applicability to other components and industries. We identify three conceptually different organizational approaches to component sharing: a coordinated approach that requires higher-level organizational echelons above the individual project, a project-by-project approach that does not, and a hybrid partially coordinated approach. We use our model to examine how the gain from the coordinated projects approach relative to the project-by-project approach varies with the number of component versions in consideration, warranty costs, complexity costs, and demand variability. Further, we use our model to highlight the risk of using simplistic heuristics to determine design sequence within a component system in a partially coordinated approach.

(Component Systems Sharing; Managing Variety; Assembled Products Design)

1. Introduction

Firms in many industries have increased the variety of their offerings in the past few decades, with a view to increasing revenues. However, variety can also increase the costs of product design, manufacturing, distribution, and after-sales support (Fisher et al. 1995, Ramdas 2003). Thus, firms struggle to offer variety cost-effectively.

Component sharing—using the same version of a component across multiple products—is an approach adopted by many assembled-product manufacturers in industries as diverse as computers, toys, and automobiles as a means to achieve high variety at low cost. For example, auto companies have implemented component sharing within their product lines both by carrying over component versions from one model year to the next, and by using a component version on multiple car models in any model year. The key questions that arise in developing a component-sharing strategy for assembled products are:

(A) How should individual products in the firm’s product line be differentiated from competitor...
products and from one another? Which component types should be shared across models, and which should be used as differentiators?

(B) What underlying product architecture should be used to support the product line?

(C) Once these high-level product line decisions have been made, how many and which versions of each component type should be offered to support the entire line, and which component versions should each model use?

Questions A and B above are clearly strategic; they define how firms choose to compete in the marketplace. While C addresses a more tactical issue, the trade-offs involved in resolving this question are often complex, and the gains from better decision making substantial. We focus on C, and assume that product-line level variety, and which components to differentiate on, are specified. This hierarchical approach is representative of the decision process used in the automobile and other assembled-product industries.

In addressing Question C, we focus on components (or features) that do not significantly affect consumers' perceptions about product differentiation, as long as they meet certain minimum performance requirements—for example, spring-clip terminals vs. sturdier binding-post terminals on a speaker, or the level of liability against theft for a credit card. For such components, a component version can be used on multiple products provided it meets the most stringent performance requirements in the set. Such downward substitution, where a “better than adequate” component version is used on some products, saves on fixed costs of design, tooling, manufacturing support, distribution, and after-sales support, but often incurs additional variable costs. As an example, using a better than adequate terminal on a speaker would save on fixed costs but incur additional materials cost. Our model is inappropriate for components that strongly influence perceived differentiation, for which sharing can result in products that seem too similar—for example, body panels of a speaker, or the breadth of stores accepting a credit card.

In focusing on less-differentiating components, we use automotive braking systems as our primary example. We simultaneously consider all of the component types that comprise a component system (for example, an automobile's braking system is comprised of pedal, booster, master cylinder, brake rotors, and brake calipers, that work together to stop it) and determine which versions of each component type should be introduced to support a defined product line and which versions should be used by each model in the line.

We chose to illustrate our modeling approach on braking systems for two reasons. First, based on our discussions with auto company executives, this domain is an excellent example of components with a relatively low impact on perceived differentiation, and it offers substantial potential cost savings from components sharing. For example, a senior Ford executive told us that choosing appropriately between sharing and designing a new braking system component could reduce that component's cost contribution in a vehicle by up to 20%. Second, the braking system is about middle of the scale in terms of the inherent complexity of design interactions, both among components within the braking system, and between the braking system and other automotive systems. Parts such as tires rank at the low end of the complexity scale, whereas engines rank at the high end. The level of complexity of braking systems is high enough so that deciding how and to what extent to share different braking system components across vehicles in a firm's product line is not a trivial problem. We came to this conclusion after speaking to several design managers and brakes design engineers at General Motors and Ford. Because the trade-offs are known but complicated, modeling is very appropriate.

We first examine in §2 the organizational and informational factors that influence components-sharing decision making. We argue that organizational echelons above the individual project level, such as product platform teams or functional area leaders, enable the interproject coordination required to take a holistic, coordinated projects approach to components-sharing decisions. In the absence of such echelons, firms are likely to make component-sharing decisions on a narrower, project-by-project basis. In practice, due to the organizational difficulties in creating coordination, component system design is sometimes done via a partially coordinated approach, where some decisions
are coordinated, while others are made on a project-by-project basis. We posit that in addition to organizational requirements, making components-sharing decisions also requires access to information on what component versions are available.

After a brief review of the mechanics of braking and the braking system design process in §3, we develop in §4 a modeling framework for component-sharing at the component system level that enables us to analyze component-sharing decisions under different organizational regimes. A major challenge in modeling component sharing at the system level is that the performance of a component system is often a complex function of its components. For example, the braking torque of an automobile is a complex function of the design parameters of the individual braking components. This creates system-to-product feasibility constraints—System torque must meet each car’s stopping requirements. In addition, there are component-to-product feasibility constraints for individual components—e.g., a car’s front brake must fit within its wheel. Further, there are interactivity constraints among components in a system, because only certain combinations of component versions of the different types can work together. We develop a model that captures these constraint types for the automotive braking system. The model objective is to select a components-sharing strategy for all braking system component types to minimize total fixed and variable costs subject to the relevant design constraints and organizational regime. A powerful characteristic of our modeling approach is that we do not specify engineering design equations, rather, only the resulting constraints. For this reason, our model is easily translatable to component systems design for other assembled products.

We develop an efficient solution procedure to our model that combines Lagrangean relaxation with a Lagrangean heuristic to obtain good feasible solutions. We illustrate our approach on a realistic problem for the auto industry and test it on much larger problems that might occur in other industry settings. We then adapt our modeling framework to reflect different organizational regimes and estimate the benefits from taking the coordinated projects approach to components sharing over the more traditional project-by-project approach, which requires less coordination. We find that these benefits are greatest when there are many component versions in consideration, as with few versions coordination is implicitly achieved even in the traditional approach. These benefits are also greater when component proliferation increases complexity and warrantee costs. We also find that the gains from the coordinated approach do not vary systematically with the underlying variability in car model demand volumes, and explain why. In addition, we compare the performance of a partially coordinated approach, where some decisions are coordinated and others are not, to that of the coordinated approach and the project-by-project approach. Finally, §5 contains concluding remarks.

Our work is related to Fisher et al. (1999), who model the trade-off involved in sharing a single component type, the automotive front brake. In their model, savings in fixed costs accruing from downward substitution are weighed against the incremental variable costs due to using overspecified component versions on some cars. They used this model to develop several testable hypotheses about components sharing that they verified using data from actual practice. While this model helped build intuition about components-sharing decisions, its applicability is limited by the fact that a car’s braking performance is in fact determined by the entire braking system, not just its front brake. The model we present in this paper addresses precisely this issue. Rutenberg (1971) has considered component sharing at the component system level, but in a model with narrowly defined component interactions. Gupta and Krishnan (1999) have also considered component sharing at the system level. In their model, the interaction between component types is limited to sharing a supplier. They do not model any design interactions such as system feasibility or interactivity constraints.

Several authors (e.g., Dobson and Yano 1995, Morgan et al. 2001, Raman and Chhajed 1995, Ramdas and Sawhney 2001) have examined the higher-level issue of how much product line variety to offer. Others (Ulrich 1995, Baldwin and Clark 1998, Robertson and Ulrich 1998) discuss architectural decisions that often provide the framework within which
components-sharing decisions are made. Krishnan and Ulrich (2001), Ramdas (2003), and Yano and Dobson (1998) review these research streams.

2. Organizational and Informational Factors that Influence Components Sharing

Sharing components involves either choosing to design a component version for use in multiple concurrent design projects or reusing a component version, either as is or with modifications. We believe that a firm’s approach to component system design and sharing reflects the organization of its design function and the availability of relevant information. Designing components for use in multiple concurrent design projects requires an organizational structure that allows coordination among, not just within, individual projects. For example, many car companies seek to achieve this type of coordination via an organizational echelon described as a product platform, which encompasses several related individual car projects.

Senior design executives at Ford indicated to us that the interpretation of such platforms has evolved over time. In the past, car projects within a platform were required to share a core set of “platform components”—often including the chassis and drive train, and were encouraged to share other non-platform component types as well, such as braking system components. Today, car projects within a platform are required to share certain aspects of the production process, known as “fixed points.” As long as these fixed points—such as the method of insertion of a particular component in assembly—remain unchanged, the components themselves need not be standardized, although sharing of many types of components is encouraged within a platform. In either interpretation of a platform, the product platform echelon facilitates sharing by allowing coordination across concurrent design projects. For some component types, sharing may even be possible across platforms. This type of sharing is facilitated organizationally when different platform groups report to common VPs for platforms engineering and product development.

Designing component versions for use within and across platforms can also be facilitated by having component designers on individual projects report to functional leaders for the different component systems. However, our discussions with auto industry executives reveal that coordination via platform echelons is preferable because functional leaders often do not have a good understanding of the holistic needs of individual car projects.

Given the difficulties in coordinating component design across multiple projects, some companies choose to coordinate decisions on some component types and use a decentralized approach for others. Important issues designers grapple with are for which types of components coordinated decision making is most appropriate, and how much is lost by using a partially coordinated approach rather than full coordination.

In deciding whether to design new component versions (or modify existing ones) for use in multiple projects, designers also need access to information on all available and potential component versions and their cost structures. In the auto industry, car companies now have databases of existing component versions that can be accessed by designers. Information on potential component versions being considered by particular teams is typically less well documented. Figure 1 summarizes the organizational and informational factors described above in the auto industry context.

In practice, companies both reuse component versions over time and share among multiple models offered concurrently, with varying degrees of supporting organizational infrastructure and information access. As an example from the auto industry, Figure 2 shows the use of front brake rotors in General Motors’ product line in the year 2000. Figure 3 shows all General Motors models that used a front brake rotor that was used in the year 2000, from the year of introduction of each rotor up to year 2000. Rotors are often used on cars that differ in weight, due to downward substitution. Also, cars that use a specific rotor are not always contiguous in weight. The same holds true for other braking system components.

Data source: an automotive research company.
Figure 1  Organizational and Informational Factors that Influence Components-Sharing Decisions

Figure 2  Usage of All Front Brake Rotors in GM's Year 2000 Product Line

Note. Three car models with sales volumes below 500 units were excluded.
these two extremes. Finally, we model a hybrid, partially coordinated approach where some decisions are made jointly across projects, while others are made on a project-by-project basis.

**Coordinated Projects Approach**

We assume that each individual project team is aware of all existing component versions for all braking system components, as well as new component versions that might be under consideration by any individual project team. While we consider all component versions available across platforms, the model can be limited to component versions within a platform. We assume that all design teams are aware of the fixed and variable costs of all component versions, and of vehicle sales volumes. We assume that teams decide jointly on which component versions to design and what versions each car will use, so as to minimize total fixed and variable costs, subject to the relevant constraints.

For each component type, the product line must be partitioned into subsets such that vehicles in each subset use the same component version. This problem is difficult to solve because the lowest-cost feasible partition of the product line for a specific component—say master cylinders—often differs from that for other components, e.g., brake rotors or boosters. Further, component decisions cannot be made independently due to system-to-product feasibility and component-interactivity constraints. We found it useful to group component versions into 5-tuples, each representing a braking system, comprised of a rotor, caliper, booster, master cylinder, and pedal, that satisfy interactivity constraints. Whether or not a 5-tuple is feasible for a particular car is a function of system-to-product feasibility, and component-to-product feasibility for its comprising component versions.

**Formulation of Coordinated Projects Problem.**

**Sets**

<table>
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<tr>
<th>Symbol</th>
<th>Description</th>
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<td>( J )</td>
<td>Car models in the product line</td>
<td>( j )</td>
</tr>
<tr>
<td>( A )</td>
<td>Component versions of all component types</td>
<td>( a )</td>
</tr>
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</table>

**Parameters**

- \( V_{kj} \) Variable cost associated with using component 5-tuple \( k \) on car model \( j \)
- \( F_a \) Fixed cost of introducing component version \( a \)

**Variables**

- \( Y_a \) Indicates whether component version \( a \) is designed \((Y_a = 1)\) or not \((Y_a = 0)\)
- \( Z_k \) Indicates whether 5-tuple \( k \) is designed \((Z_k = 1)\) or not \((Z_k = 0)\)
- \( W_{kj} \) Indicates whether 5-tuple \( k \) is used on car \( j \) \((W_{kj} = 1)\) or not \((W_{kj} = 0)\)

Minimize \( \sum_{a \in A} F_a Y_a + \sum_{k \in K} V_{kj} W_{kj} \)

subject to

\[
W_{kj} \leq Z_k \quad \forall j \in J, k \in K \\
\sum_{i \in j} W_{ki} = 1 \quad \forall j \in J
\]

\[
Z_k \leq Y_a \quad \forall a \in A, k \in K_a
\]

\[
Z_k, W_{kj}, Y_a \in \{0, 1\} \quad \forall j \in J, k \in K, a \in A.
\]

This problem shares some features with the simple plant-location problem (SPLP). Designing a component 5-tuple is analogous to opening a plant, and assigning a 5-tuple to a car is analogous to assigning a plant to a customer. Constraints (1) ensure that a 5-tuple can be assigned to a car only if it has been designed. Constraints (2) ensure that each car is assigned a feasible braking system. Similarly in the SPLP, a plant can be assigned to a customer only if it is open, and each customer must be assigned a plant. The key difference between the above formulation and the SPLP is that in the SPLP there is a fixed cost for opening each plant, whereas above, the cost of designing each 5-tuple is the sum of the costs of designing its components. If a component is
3. The Braking System Design Process

We learned about braking system design by meeting with several automotive design managers and brake engineers. In essence, a braking system is a hydraulic system that converts foot pressure applied at the pedal into a much higher braking pressure applied at the wheels via the braking system components. The pressure applied at the wheels forces stationary brake components to rub against rotating components, thus converting the kinetic energy of a moving car into heat energy via friction.

Automotive braking system design is initiated only after vehicle design has been broadly specified via “system-level parameters” such as vehicle weight, top speed, and stopping distance. Given these inputs, the components of the braking system must be designed so as to provide adequate torque to stop a car from top speed within the desired stopping distance. Braking system design parameters like rotor radius, desired pedal force, and area of the caliper pistons and master cylinder piston are manipulated to meet this end.

All braking components are designed for “maximum loading” conditions. For example, the brake pedal should not break if the driver steps exceptionally hard on it in a panic stop. These conditions, together with space and layout issues, result in component-to-product feasibility constraints. As mentioned earlier, the braking torque requirement constitutes a system-to-product feasibility constraint. Several component interactivity constraints also arise. For example, the hydraulic ratio (ratio of areas of master cylinder and caliper pistons) must lie within prespecified limits to eliminate excessive pedal “travel,” which could cause the pedal to hit the floor of the car.

4. Modeling Component Systems Sharing for Automotive Braking Systems

We first model two conceptually different approaches to component sharing, reflecting whether or not a firm has put in place one or more higher organizational echelons above the single-project level, to allow joint decisions among multiple individual new product projects on what component versions of each type to offer. We refer to the case where such echelons do exist as the coordinated projects approach and the case where they do not as the project-by-project approach. While in practice a firm may implement platform echelons and higher coordinating echelons to varying degrees, resulting in lesser or greater coordination, we find it useful to distinguish conceptually between
these two extremes. Finally, we model a hybrid, partially coordinated approach where some decisions are made jointly across projects, while others are made on a project-by-project basis.

Coordinated Projects Approach
We assume that each individual project team is aware of all existing component versions for all braking system components, as well as new component versions that might be under consideration by any individual project team. While we consider all component versions available across platforms, the model can be limited to component versions within a platform. We assume that all design teams are aware of the fixed and variable costs of all component versions, and of vehicle sales volumes. We assume that teams decide jointly on which component versions to design and what versions each car will use, so as to minimize total fixed and variable costs, subject to the relevant constraints.

For each component type, the product line must be partitioned into subsets such that vehicles in each subset use the same component version. This problem is difficult to solve because the lowest-cost-feasible partition of the product line for a specific component—say master cylinders—often differs from that for other components, e.g., brake rotors or boosters. Further, component decisions cannot be made independently due to system-to-product feasibility and component-interactivity constraints. We found it useful to group component versions into 5-tuples, each representing a braking system, comprised of a rotor, caliper, booster, master cylinder, and pedal, that satisfy interactivity constraints. Whether or not a 5-tuple is feasible for a particular car is a function of system-to-product feasibility, and component-to-product feasibility for its comprising component versions.

Formulation of Coordinated Projects Problem.

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<td>Variable cost associated with using component 5-tuple $k$ on car model $j$</td>
</tr>
<tr>
<td>$F_a$</td>
<td>Fixed cost of introducing component version $a$</td>
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</tr>
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<td>Indicates whether 5-tuple $k$ is used on car $j$ ($W_{kj} = 1$) or not ($W_{kj} = 0$)</td>
</tr>
</tbody>
</table>

Minimize

$$\sum_{a \in A} F_a Y_a + \sum_{k \in K} V_{kj} W_{kj}$$

subject to

$$W_{kj} \leq Z_k \quad \forall j \in J, k \in K$$

$$\sum_{k \in K} W_{kj} = 1 \quad \forall j \in J$$

$$Z_k \leq Y_a \quad \forall a \in A, k \in K_a$$

$$Z_k, W_{kj}, Y_a \in \{0, 1\} \quad \forall j \in J, k \in K, a \in A.$$ (4)

This problem shares some features with the simple plant-location problem (SPLP). Designing a component 5-tuple is analogous to opening a plant, and assigning a 5-tuple to a car is analogous to assigning a plant to a customer. Constraints (1) ensure that a 5-tuple can be assigned to a car only if it has been designed. Constraints (2) ensure that each car is assigned a feasible braking system. Similarly in the SPLP, a plant can be assigned to a customer only if it is open, and each customer must be assigned a plant. The key difference between the above formulation and the SPLP is that in the SPLP there is a fixed cost for opening each plant, whereas above, the cost of designing each 5-tuple is the sum of the costs of designing its components. If a component is
used in two, 5-tuples, its design cost is counted only once. Constraints (3), together with the form of the objective function, impose this restriction. As in the SPLP, the \( W_{ij} \) variables in this formulation take on integral values even if the corresponding integrality constraints are dropped. This formulation is particularly attractive relative to a formulation that models components only at the individual level if the number of feasible 5-tuples is small relative to the number of possible combinations of the underlying components, as is typical in practice.

If Constraints (2) and (3) above are relaxed, the remaining problem can be solved by inspection. The Lagrangean relaxation (LR) bound obtained by relaxing these constraints equals the linear programming (LP) bound for this problem, as the remaining problem has the integrality property. We heuristically solve the coordinated projects problem via Lagrangean relaxation together with a Lagrangean heuristic, both described in Appendix.

We first tested this method on a realistic size problem for the auto industry, motivated by General Motors' product line in the year 2000. In 2000, GM offered 32 different car models, with average sales per model of 70,000 units, and coefficient of variation of sales of 0.9. We created a problem of this size for which we simulated normally distributed demand with mean and standard deviation based on the company's year 2000 line, adjusting demand upwards to reflect a two- to three-year planning cycle typical for braking components. In our problem, the number of component versions of each type in consideration was set similar to the number offered by GM in its year 2000 product line. With a total of 64 component versions in the consideration set \( A \), we simulated realistic component interactivity, component-to-product feasibility, and system-to-product feasibility constraints by limiting the number of feasible 5-tuples, component-to-car assignments and 5-tuple-to-car assignments. We also generated fixed costs and unit costs for the component versions of different types, similar in magnitude to those seen in the industry, with higher unit costs for more "heavy-duty" component versions that were just adequate for heavier cars. Figure 4 pictorially depicts the problem inputs. Notice that we do not require component versions to be feasible on cars that are contiguous in weight, unlike Fisher et al. (1999) and Gupta and Krishnan (1999). We found that on this realistic problem our solution method resulted in a gap of 0.8% between the best feasible solution and the best Lagrangean relaxation lower bound in 500 subgradient iterations taking 26 CPU seconds on a Pentium 4 processor. We also tested our solution method on much larger problems that might occur in other industrial settings, and found that it performed very well. Table 1 contains representative results.

**Project-by-Project Approach**

We assume as before that each individual project team is aware of all existing and potential component versions of all components, and that all design teams are aware of fixed and variable costs of all components, and sales volume for all models. However, unlike previously, we assume that each team decides independently on which component versions to use to meet its individual car project's needs.

**Independent Subproblem Formulation to Select All Components for the \( j \)th Car Project.**

\[
\text{Minimize } \sum_{a \in A} F_a Y_a + \sum_{k \in K} V_{kj} W_{kj}
\]

subject to Equation (3) and the constraints from Equations (1), (2), and (4) of the coordinated projects problem that are relevant to model \( j \).

Let \( W_{ij}^{*} \) denote the optimal value of \( W_{ij} \) in the independent subproblem for the \( j \)th car project for each \( k \in K, j \in J \). Also, let indicator \( Y_a^* \) denote whether component version \( a \) was used in the optimal solution for any of the individual car projects. Then the total cost over all car projects using a project-by-project approach is \( \sum_{a \in A} F_a Y_a^* + \sum_{k \in K} \sum_{j \in J} V_{kj} W_{kj}^* \). Notice that we count the fixed cost for each component version only
once, although different car project teams may have independently decided to design the same component version. In practice, teams will occasionally duplicate design effort and develop identical component versions. This would further worsen the performance of the project-by-project approach relative to the coordinated approach.

In estimating the fixed cost of introducing a new component version, we included the costs of designing the component version, engineering and validating the design, and designing and building tooling. Increasing component versions also increases manufacturing complexity cost. Also, in recent years, assembled product manufacturers, e.g., auto companies, have identified increased warranty costs as a major, underestimated downside of component proliferation—With low volumes, there is less opportunity to improve processes and remove glitches over time. While the negative impact of component proliferation on complexity costs and warranty costs is hard to estimate, conceptually it can be viewed as increasing the fixed cost of a new component version. We will analyze this downside of proliferation via sensitivity analysis on fixed costs.

<table>
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<tr>
<th>Problem no.</th>
<th>Number of end products</th>
<th>Total number of versions over all component types</th>
<th>Number of feasible component systems</th>
<th>% gap between best Lagrangean lower bound and best feasible solution</th>
<th>Time taken in CPU seconds*</th>
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<td>727</td>
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*Note: *based on 250 subgradient iterations on a Pentium 4 processor.
To gain intuition on when the coordinated projects approach is most valuable, we compared this approach with the project-by-project approach over a large number of test problems generated in a structured fashion. Starting with our test problem with size, demand, and cost parameters representative of the auto industry, we generated test problems by varying three important parameters that affect design decisions. The first, $F^w$, is a multiplier applied to the fixed cost of each new component version to capture the impact of complexity and warranty costs. We assume at first that $F^w$ is constant across all component versions. The second parameter, $N_c$, denotes the total number of component versions in consideration in set $A$. In practice, a history of low coordination across individual projects often results in an explosion in the number of component versions. The third parameter, $cv$, represents the coefficient of variation of car demand. We considered $F^w = 1, 2, 3, \text{ and } 4$ (where $F^w = 1$ gives us the design costs in the base problem); $N_c = b, 1.5b, \text{ and } 2b$ where $b$ equaled the total number of component versions in the base problem, and $cv = 0.1, 0.5, \text{ and } 0.9$. Starting with the base problem, we generated 200 test problems for each combination of $F^w, N_c$, and $cv$. We first computed, in each case, the average improvement in objective value from using the coordinated projects approach over the project-by-project approach by comparing the best solution obtained in the coordinated approach with the optimal solution to the project-by-project approach, which was solved manually.

We expected deterioration in performance of the project-by-project approach relative to the coordinated projects approach with increasing complexity and warranty costs (i.e., higher $F^w$) due to higher penalty for myopic decision making. We expected poorer relative performance for the project-by-project approach for problems with a greater number of component versions, i.e., with higher $N_c$. This is because with many component versions, there is a greater chance that the optimal component choice for one car will differ from that for another, unlike the case in which there are few component versions and coordination is implicitly achieved even in the project-by-project approach.

We also expected deterioration in performance of the project-by-project approach relative to the coordinated projects approach with an increase in coefficient of variation of demand, $cv$, for the following reason. For any level of mean demand, with higher $cv$ there would be more cars with either very low or very high demand. In the project-by-project approach, each car design team would independently select a feasible braking system with the lowest sum of fixed and variable costs, even if demand were very low. This would lead a team to design a braking system that is "just adequate" over using a better than adequate system, even if the latter would need to be designed in any case, for a heavier car. Doing this ignores potential savings that would accrue if the incremental variable cost from using the better than adequate system were lower than the fixed cost for the just adequate system. Of course, with high $cv$ we would also see some car models with much higher than average demand, in which case this problem would not arise.

Based on our sensitivity analyses we found, as expected, that other things being equal, the performance of the project-by-project approach deteriorated significantly relative to the coordinated projects approach for higher values of $F^w$ and $N_c$. Unexpectedly, we found that other things being equal, the relative performance of the project-by-project approach could either deteriorate or improve for higher values of $cv$. We found that this happens because even in the case where a car model has much higher than average demand, the incremental variable cost from using a better than adequate system that is ideal for a heavier car can be lower than the fixed cost for the just adequate system, resulting in sharing in the coordinated projects approach. However, in this case the improvement from the coordinated projects approach is smaller than in the case where the lighter cars had below-average demand. The results of the sensitivity analysis are in Figure 5. Because $cv$ has no systematic impact, we report average performance difference across different values of $cv$. Running sensitivity analyses using our modeling framework thus helped confirm our intuition about the impact of complexity and

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5Because the realized coefficient of variation differed in each instance, we examined performance as a function of this parameter rather than the specified coefficient of variation, $cv$. 

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warranty costs and greater choice of component versions, and refined our intuition about the impact of demand variability.

In our sensitivity analyses, we also examined the solutions generated by the two approaches with a view to further understanding the implications of coordination. In Figure 6, we report differences in these solutions for varying numbers of potential component versions \(N_c = b, 1.5b, 2b\), obtained by averaging over all values of \(F^p\) and \(c_v\). In every test instance, the coordinated projects solution used fewer components versions and 5-tuples than the project-by-project solution. The percentage reduction in the number of component versions used was increasing in \(N_c\), highlighting why the relative performance of the coordinated approach improves with larger \(N_c\). While most of the component versions used in the coordinated projects solution were common to the project-by-project solution, some were unique. In the coordinated approach, a component version with high design costs that was feasible for multiple products could dominate component versions with lower design costs that could serve only one product, whereas the project-by-project approach would ignore the possibility of a single component version serving multiple products. On average, only a third of the 5-tuples offered in the coordinated solution were common to the project-by-project solution, suggesting that the same underlying component versions were combined more effectively. Although the proportion of component versions in the coordinated solution that were common to the project-by-project solution did not change much with the number of component versions in consideration, there was a reduction in the proportion of 5-tuples in the coordinated solution that were common to the project-by-project solution, for problems with more component versions. Thus, for problems with more component versions, the coordinated projects approach was more likely to select system configurations ignored in the project-by-project approach. This further explains why the relative performance of the project-by-project approach deteriorates as the number of component versions in consideration increases.

**Partially Coordinated Approach**

This approach falls between the organizational extremes of the coordinated approach and the project-by-project approach. We assume as before that each individual project team is aware of all existing and potential component versions for all braking system components across all platforms, and that all design teams are aware of the fixed and variable costs of these component versions. However, we assume that for some braking system components, all teams decide jointly on which component versions to use to support the entire product line, whereas for other braking system components, each team decides independently on which component versions to use to meet
its individual car project's needs. In what follows, we assume first that decisions for rotors, calipers, master cylinders, and boosters are made jointly, and that decisions for pedals are made independently. In practice, the coordinated decisions are often made earlier than the independent decisions. This is done by ensuring in the coordinated decision-making process that for the specific component versions of jointly managed components chosen for a car, there would be at least one feasible pedal version. So, for example, for a given car, a rotor, caliper, master cylinder, and booster that would require a pedal with excessive "travel" to provide the needed torque requirements would be eliminated in the coordinated design stage. Let $A^p \subset A$ denote the set of all pedal versions, indexed by $p$. Let $A_k = \{r_k, e_k, m_k, b_k, p_k\}$ denote the set of component versions—rotor, caliper, master cylinder, booster, and pedal—that comprise $k$-tuple $k$ and let $a_k$ denote any element of $A_k$. Further, let $V_{a_k} = v_{a_j} + v_{m_j} + v_{b_j} + v_{p_j}$, where $v_{a_j}$ denotes the variable cost associated with using rotor version $r_k$ on car $j$, etc. We model below a coordinated master problem and a set of independent subproblems.

**Coordinated Master Problem Formulation.**

Minimize $\sum_{a \in A - A^p} F_a Y_a + \sum_{k \in A} W_{a_k} V_{a_k}$ subject to Constraints (1) and (2) of the coordinated projects problem formulation and the subset of Constraints (3) and (4) for which $a \in A - A^p$, i.e., which excludes all pedals.

Rotor, caliper, master cylinder, and booster assignments for each car $j$ are based on the optimal solution to the above master problem. For each $k, j$, and $a_k \in A_k$, let $T_{a_k}^{MP}$ be an indicator denoting whether or not component $a_k$ is assigned to car $j$ in the optimal solution to the master problem. Next, uncoordinated subproblems are solved as follows for each car project $j$ to determine the pedal assignments that complete the assignment of all braking system components to each car.

**Independent Subproblem Formulation to Select Pedal for the $j$th Car Project.**

Minimize $\sum_{a \in A^p} F_a Y_a + \sum_{k \in A} W_{a_k} V_{a_k}$ subject to

$$\sum_{k \in A} W_{a_k} = 1$$

and the constraints from (1), (3), and (4) of the coordinated projects problem formulation that are relevant to product $j$.

Let $MP^*$ denote the optimal objective function value of the coordinated master problem, $W_{a_k}^{SP^*}$ denote the optimal value of $W_{a_k}$ in the independent subproblem for the $j$th car project for each $k \in K, j \in J$; and $Y_{a_k}^{SP^*}$ denote whether or not pedal version $a \in A^p$ was designed in the optimal solution to any of the independent subproblems. Then the total cost in the partially coordinated approach can be computed as:

$$MP^* + \sum_{a \in A^p} F_a Y_{a_k}^{SP^*} + \sum_{k \in A} W_{a_k}^{SP^*} V_{a_k}$$

For our test problem reflecting auto industry size and parameters, we compared the partially coordinated approach with the coordinated approach and the project-by-project approach, first with pedals designed last and then with master cylinders designed last. Partial coordination resulted in lower total costs than the project-by-project approach. However, partial coordination via designing pedals last was only slightly better overall than via designing master cylinders last, even though design costs for master cylinders were much higher than for pedals. If we further increased the design cost for master cylinders (this was done by applying the multiplier $F^W$ to this component type only), the performance of the partially sequential approach with master cylinders designed last deteriorated appreciably relative to the coordinated approach (see Figure 7). However, we found that this deterioration in performance could be reduced by relaxing the stringency of the component-to-product feasibility constraints for master cylinders. An important learning is that simplistic rules of thumb such as "design the component type with cheapest design cost last" are not adequate for managing component sharing in a complex design process. While relative design costs for different component types are important, other factors such as the
relative stringency of component-to-product feasibility constraints and the impact of different component types on interactivity constraints need also be considered. The modeling approach we have developed provides a way to navigate such sophisticated comparisons.

Components that can be cheaply custom tailored to each product to fit early choices made for other components are good candidates to design last. Such components will have low design costs and ample “design slack” to absorb constraints imposed by early decisions on other system components and still deliver the needed system performance.

Interestingly, despite increasing the degree of coordination needed across projects, it is possible to construct theoretical examples in which the partially coordinated approach can actually perform worse than the project-by-project approach. This happens because coordinating across projects on some component types while ignoring the cost of the later-designed component types can constrain the choice of the latter to expensive alternatives, whereas the project-by-project approach would instead consider costs over all component types, albeit without coordinating decisions across projects. However, in our test problems driven by reasonable cost data, we did not find this to happen.

5. Concluding Remarks
We developed a very general modeling approach for component sharing at the component systems level for components that do not significantly impact consumers’ perceptions about product differentiation. Our approach is significantly more realistic than previous approaches that either model component sharing only for individual components in a system, or fail to capture most of the complexities while modeling systems-level sharing. We identify three conceptually different types of constraints in system-level component sharing: component-to-product feasibility, system-to-product feasibility, and component interactivity constraints. We illustrate our approach on a problem representative of braking systems design in the auto industry, and test it on much larger problems that might arise in other settings.

We identify organizational and informational factors that influence component-sharing decisions and are able to identify three conceptually distinct organizational regimes:

- The presence of organizational echelons higher than the individual project level facilitates joint decision making across individual car projects on what component versions to design and use to support the entire product line, leading to a coordinated projects approach.
- The absence of such echelons results in component-sharing decisions being made on a project-by-project basis.
- For a component system, some decisions may be coordinated and others made independently later on, resulting in a partially coordinated approach.

We use our model to examine the benefits of the coordinated projects approach relative to the project-by-project approach, for different levels of costs, number of component versions in consideration, and demand variability. We find that these benefits are increasing in design costs, complexity costs, warranty costs, and in the number of potential component versions, but do not vary systematically with demand variability. The coordinated projects approach uses fewer component versions and fewer component systems than a project-by-project approach. Although many specific component and system versions may be common across the two approaches, overall, fewer versions are combined more effectively to generate lower costs.

We also use our model to examine how partial coordination compares against full coordination or an
uncoordinated, project-by-project approach. Thus, we are able to show how sharing decisions are limited by the organizational and informational infrastructure within which they are made. In addition, we highlight the risks of using simplistic heuristics to determine which component types in a system can be designed later.

Estimating the benefits of switching to a coordinated projects approach is important because creating the organizational infrastructure to support such an approach can itself be quite costly—Therefore, a firm can use the estimated gains under different scenarios to determine whether reorganization is worthwhile. Interestingly, we find that even disregarding coordination cost, coordination is not always beneficial. If the downstream constraints imposed by partial coordination on some components upstream outweigh the fixed cost savings, the partially coordinated approach can perform worse than the uncoordinated, project-by-project approach.

While we illustrated our modeling approach for automotive braking systems, it is equally applicable to systems-level component sharing for other physical assembled products and also to software products, information products, or financial services that are comprised of modular systems. For the latter product categories, the variable costs of using a better than adequate component version are often diseconomy costs to the customer: e.g., increased service response time due to using a more detailed than needed credit-checking process in a financial service. That we do not place any restrictions on the nature of the fixed and variable cost functions associated with components and system systems, or the nature of the design constraints, provides a wide platform for modeling diverse applications.

Our modeling framework is appropriate at the stage in development where high-level decisions about differentiating product characteristics and architecture have already been made. A limitation is that we focus on a snapshot of a firm’s variety management decisions, while these in fact evolve continually over time. Future research should examine the dynamics of component sharing decisions.

The data requirements for implementing this type of modeling are fairly modest as firms typically track demand and cost information, and engineers need to be aware of the specifics of the design constraints they face in order to develop new products. That our modeling approach does not require specification of engineering design equations, rather only the resulting constraints, greatly reduces information requirements. We believe that a greater challenge in implementing such models is that designers may view them as a distraction from the main design task, as noted also by Krishnan et al. (1999). To overcome this problem, future models must be integrated into existing design tools. Engineers may also be uncomfortable providing the inexact demand and engineering cost estimates required for such models—a problem that would be mitigated by more reliable data gathering and building confidence in the power of such models.

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Appendix. Solution Procedure for System-Level Formulation to the Coordinated Projects Problem

Solution Procedure for Each Lagrangean Subproblem: Let $\mu_j$ and $\lambda_{ai}$ denote the Lagrangean multipliers associated with Constraints (2) and (3) in the coordinated projects formulation; let $A_i = \{p_i, d_i, r_i, m_i, b_i, p_i\}$ denote the set of rotor, caliper, master cylinder, brake, and pedal versions comprising a tuple $k$, and let $a_i$ denote any component version in $A_i$. The Lagrangean subproblem obtained is:

$$\text{Minimize} \sum_{a, \lambda} \left( f_{ij} - \sum_{k \in \lambda} \lambda_{ai} \right) Y_{ij} + \sum_{j \in \lambda} \left( V_j - \mu_j \right) W_j + \sum_{k \in \lambda} \sum_{a \in \lambda} \lambda_{ai} Z_k + \sum_{j \in \lambda} \mu_j$$

subject to (1) and (4).

Let $Y_j^\mu$, $Z_k^\mu$, $W_j^\mu$ denote the optimal values of $Y_j$, $Z_k$, $W_j$ in the Lagrangean subproblem. Set $Y_j^\mu = W_j^\mu = Z_k^\mu = 0 \ \forall j, k, \mu$. Then, for $a \in A_i$, if $f_{ij} - \sum_{k \in \lambda} \lambda_{ai} < 0$, set $Y_j^\mu = 1$. For $k \in K$, if $\sum_{a \in \lambda} \lambda_{ai} (V_k - \mu_k) + \sum_{a \in \lambda} \lambda_{ai} Z_k < 0$, set $Z_k^\mu = 1$ and $W_j^\mu = 0$.

This gives us the optimal solution to the Lagrangean subproblem.

Lagrangean Heuristic: This heuristic returns a feasible solution to the coordinated projects problem. We first reorder all $j \in J$ by descending sales volume per model. Let $Y_j^\mu$ denote the value of $Y_j$ returned via the Lagrangean heuristic, and indicator $X_j^\mu$ denote whether or
not component version \( a \) is assigned to car \( j \) in the Lagrangean heuristic. To construct a feasible solution, we take advantage of the fact that some components have already been designed (i.e., \( Y_{a}^* = 1 \) for some \( a \)) in the Lagrangean subproblem.

Step 1. Set \( j = 1 \). While \( 1 \leq j < N \), pick \( k \in K^j_j \), \( A_k = \{ \tau_k, c_k, m_k, b_k, p_k \} \), such that

\[
k = \text{Arg min} \left\{ \tau_k (1 - Y_{a}^*) + f_k (1 - Y_{a}^*) + F_{a} (1 - Y_{a}^*) + F_{a} (1 - Y_{a}^*) + V_{a} \right\}.
\]

For all such 5-tuples \( k \), set \( Y_{a}^* = 1 \) for all component versions \( a \in A_k \). Set \( j = j + 1 \).

Step 2. Set \( j = 1 \). While \( 1 \leq j < N \), pick a single \( k \in K^j_j \), \( A_k = \{ \tau_k, c_k, m_k, b_k, p_k \} \) such that

\[
k = \text{Arg min} \left\{ \tau_k + f_k + F_{a} + F_{a} + V_{a} \right\}
\]

\[
Y_{a}^* = Y_{a}^* = Y_{a}^* = Y_{a}^* = Y_{a}^* = 1.
\]

Set \( X_{a,j}^* = X_{a,j}^* = X_{a,j}^* = X_{a,j}^* = X_{a,j}^* = 1 \).

Step 3. For each \( a \in A \), if \( \sum_{j \in C} Y_{a}^* = 0 \), set \( Y_{a}^* = 0 \).

This gives us a feasible solution to the coordinated projects problem starting with the optimal solution to the Lagrangean subproblem.

References


